1) Which of the following integrals are improper? Why?

    a) $\displaystyle\int_{1}^{2} \frac{1}{2x-1}\,dx$

    b) $\displaystyle\int_{0}^{1} \frac{1}{2x-1}\,dx$

    c) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}\,dx$

    d) $\displaystyle\int_{1}^{2} \ln(x-1)\,dx$

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

2) $\displaystyle\int_{1}^{\infty} \frac{1}{(3x+1)^2}\,dx$

3) $\displaystyle\int_{-\infty}^{0} \frac{1}{2x-5}\,dx$

4) $\displaystyle\int_0^\infty \frac{x}{(x^2+2)^2}\, dx$

5) $\displaystyle\int_{-\infty}^{-1} e^{-2t}\, dt$

6) $\displaystyle\int_{-\infty}^\infty \frac{x\, dx}{1+x^2}$

7) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\, dx$

8) $\displaystyle\int_{-\infty}^{\infty} e^{-|x|}\, dx$

9) $\displaystyle\int_{0}^{\infty} se^{-5s}\, ds$

10) $\int_{-\infty}^{6} re^{r/3}\, dr$

11) $\int_{1}^{\infty} \dfrac{\ln x}{x^3}\, dx$

12) $\int_{0}^{3} \dfrac{dx}{\sqrt{x}}$

13) $\int_{-1}^{0} \dfrac{dx}{x^2}$

14) $\int_{1}^{9} \dfrac{dx}{\sqrt[3]{x-9}}$

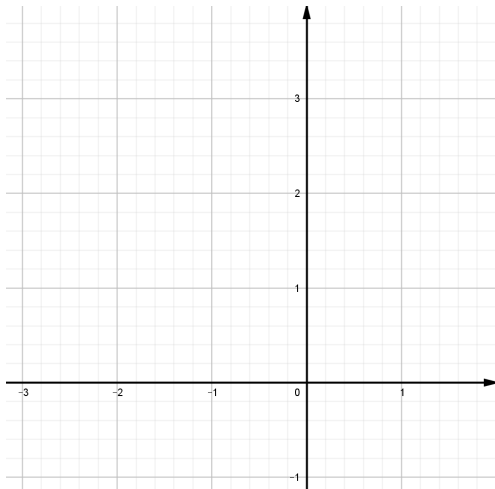15) $\int_{-2}^{3} \dfrac{dx}{x^4}$

16) $\int_{1/4}^{1} \dfrac{1}{4y-1} \, dy$

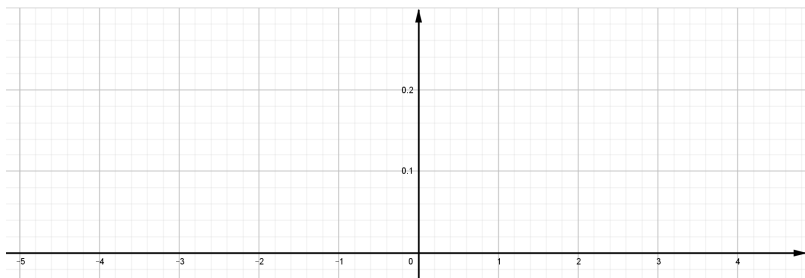17) $\int_{0}^{\pi} \sec x \, dx$

18) $\int_{0}^{4} \dfrac{dx}{x^2+x-6}$

Sketch the region and find its area (if the area is finite).

19) $S = \left\{ (x, y) \mid x \leq 1, \ 0 \leq y \leq e^x \right\}$

20) $S = \left\{ (x, y) \mid 0 \leq y \leq \dfrac{2}{x^2 + 9} \right\}$

21) A manufacturer of light bulbs wants to produce bulbs that last about 700 hours but, of course, some bulbs burn out faster than others. Let $F(t)$ be the fraction of the company's bulbs that burn out before t hours, so $F(t)$ always lies between 0 and 1.

a) What is the meaning of the derivative $r(t) = F'(t)$?

b) What is the value of $\int_0^{\infty} r(t)\, dt$? Why?